

Resoluções

Capítulo 19

Inequações exponenciais



ATIVIDADES PARA SALA

01 a) $2^x \cdot 2 + 2^x > 12 \Rightarrow 3 \cdot 2^x > 12 \Rightarrow 2^x > 2^2 \Rightarrow x > 2$

$S = \{x \in \mathbb{R} \mid x > 2\}$

b) $(2^x)^2 + 4 \cdot 2^x - 32 > 0$

Considerando $2^x = m$, tem-se:

$m^2 + 4m - 32 > 0$

$m < -8$ ou $m > 4$

$2^x < -8$ (impossível) ou

$2^x > 4 \Rightarrow x > 2 \Rightarrow S = \{x \in \mathbb{R} \mid x > 2\}$

c) $\left(\frac{1}{2}\right)^x \cdot \left(\frac{1}{2}\right)^{-2} < \left(\frac{1}{2}\right)^{-x}$

$x - 2 > -x \Rightarrow 2x > 2 \Rightarrow x > 1$

$S = \{x \in \mathbb{R} \mid x > 1\}$

d) $\left(\frac{1}{3}\right)^{x+\frac{1}{3}} \geq \left(\frac{1}{3}\right)^4 \Rightarrow x + \frac{1}{3} \leq 4 \Rightarrow x \leq \frac{11}{3}$

$S = \left\{x \in \mathbb{R} \mid x \leq \frac{11}{3}\right\}$

02 a) $(0,2)^x - 0,008 \geq 0 \Rightarrow (0,2)^x \geq (0,2)^3 \Rightarrow x \leq 3$

$D = \{x \in \mathbb{R} \mid x \leq 3\}$

b) $6^x - 6$ pode assumir qualquer valor real $\Rightarrow D = \mathbb{R}$

c) $3^x - 9 > 0 \Rightarrow 3^x > 3^2 \Rightarrow x > 2 \Rightarrow D = \{x \in \mathbb{R} \mid x > 2\}$

d) $4^x \neq 2^x \Rightarrow 2x \neq x \Rightarrow x \neq 0 \Rightarrow D = \{x \in \mathbb{R} \mid x \neq 0\}$

03 $9 < 27^{3x-1}$ e $27^{3x-1} < 27$

$2 < (3x-1) \cdot 3$ $3x-1 < 1$

$x > \frac{5}{9}$ $x < \frac{2}{3}$

$S = \left\{x \in \mathbb{R} \mid \frac{5}{9} < x < \frac{2}{3}\right\}$

04 $3^{2x} \cdot 3 - 3^{2x} - 3^{2x} \cdot 3^{-1} - 3^{2x} \cdot 3^{-2} \geq 126$

$3^{2x} \cdot \frac{14}{9} \geq 126 \Rightarrow 3^{2x} \geq 81 \Rightarrow 2x \geq 4 \Rightarrow x \geq 2$

$S = \{x \in \mathbb{R} \mid x \geq 2\}$

05 $4^x - 6 \cdot 2^x + 8 \leq 0 \Rightarrow (2^x)^2 - 6 \cdot 2^x + 8 \leq 0 \Rightarrow 2 \leq 2^x \leq 4 \Rightarrow$

$1 \leq x \leq 2$

$S = \{x \in \mathbb{R} \mid 1 \leq x \leq 2\}$



ATIVIDADES PROPOSTAS

01 a) $5x - 1 \geq 2x + 8 \Rightarrow 3x \geq 9 \Rightarrow x \geq 3$

$S = \{x \in \mathbb{R} \mid x \geq 3\}$

b) $3x > 4x - 2 \Rightarrow x < 2$

$S = \{x \in \mathbb{R} \mid x < 2\}$

c) $x^2 - 3x < -3 \Rightarrow x^2 - 3x + 3 < 0 \Rightarrow$

$S = \emptyset$

d) $-2(8x+1) \geq 16x \Rightarrow 16x \leq -16x - 2 \Rightarrow x \leq -\frac{1}{16}$

$S = \left\{x \in \mathbb{R} \mid x \leq -\frac{1}{16}\right\}$

e) $x(x+2) \leq 2x+4 \Rightarrow x^2 \leq 4 \Rightarrow -2 \leq x \leq 2$

$S = \{x \in \mathbb{R} \mid -2 \leq x \leq 2\}$

f) $2(-x-10) < -2x-6 \Rightarrow -20 < -6$

$S = \mathbb{R}$

02 a) $2^x - 2^{1-x} \geq 0 \Rightarrow 2^x \geq 2^{1-x} \Rightarrow x \geq 1-x \Rightarrow x \geq \frac{1}{2}$

$D = \left\{x \in \mathbb{R} \mid x \geq \frac{1}{2}\right\}$

b) $(0,2)^{x^2-5x} \geq (0,2)^{-6} \Rightarrow x^2 - 5x \leq -6 \Rightarrow$

$x^2 - 5x + 6 \leq 0 \Rightarrow 2 \leq x \leq 3$

$D = \{x \in \mathbb{R} \mid 2 \leq x \leq 3\}$

03 $2^x + 2^{x+1} \geq 12 \Rightarrow 2^x(1+2) \geq 12 \Rightarrow 2^x \geq 2^2 \Rightarrow x \geq 2$

Para todo $\{x \in \mathbb{R} \mid x \geq 2\}$.

04 $3^x(3+3^2) > 108 \Rightarrow 3^x > 9 \Rightarrow x > 2$

$S = \{x \in \mathbb{R} \mid x > 2\}$

05 $(3^x)^2 \cdot 3^2 - 3^x \cdot 3^3 > 3^x - 3$

Considerando $3^x = m$, tem-se:

$9m^2 - 27m > m - 3$

$9m^2 - 28m + 3 > 0 \Rightarrow m < \frac{1}{9}$ ou $m > 3$

$3^x < 3^{-2}$ $3^x > 3$
 $x < -2$ $x > 1$

$$S = \{x \in \mathbb{R} \mid x < -2 \text{ ou } x > 1\}$$

$$\boxed{06} \quad 1 < 2^{x^2-4x} < 32 \Rightarrow \underbrace{0 < x^2-4x < 5}_{(I)} \quad \underbrace{(II)}$$

$$(I) \quad x^2 - 4x > 0 \Rightarrow x < 0 \text{ ou } x > 4$$

$$(II) \quad x^2 - 4x < 5 \Rightarrow -1 < x < 5$$

$$(I) \cap (II) \Rightarrow -1 < x < 0 \text{ ou } 4 < x < 5$$

$$\boxed{07} \quad 3^{|x|} > 3^2 \Rightarrow |x| > 2$$

$$x < -2 \text{ ou } x > 2$$

$$S = \{x \in \mathbb{R} \mid x < -2 \text{ ou } x > 2\}$$

$$\boxed{08} \quad 2^0 \leq 2^{\frac{x}{3}} \leq 2^2 \Rightarrow 0 \leq \frac{x}{3} \leq 2 \Rightarrow 0 \leq x \leq 6$$

$$S = \{x \in \mathbb{R} \mid 0 \leq x \leq 6\}$$

$$\boxed{09} \quad \text{Como } e^x + 1 > 0, \text{ basta resolver } 1 - x^2 < 0.$$

$$x^2 > 1 \Rightarrow x < -1 \text{ ou } x > 1$$

$$S = \{x \in \mathbb{R} \mid x < -1 \text{ ou } x > 1\}$$

$$\boxed{10} \quad 3^{\frac{1}{x}} < 3^{-2} \Rightarrow \frac{1}{x} + 2 < 0 \Rightarrow \frac{1+2x}{x} < 0$$

$$f(x) = 1 + 2x$$

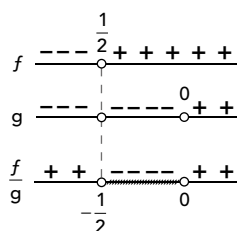
$$1 + 2x = 0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

$$g(x) = x$$

$$x = 0$$



$$S = \left\{x \in \mathbb{R} \mid -\frac{1}{2} < x < 0\right\}$$