Resoluções

Capítulo 4

Lei dos Senos e Lei dos Cossenos

ATIVIDADES PARA SALA

01 B

Sendo $\hat{A} = 30°$ e $\hat{B} = 105°$, então $\hat{C} = 45°$, logo, pela lei dos senos: $\dfrac{\overline{BC}}{\text{sen}\hat{A}} = \dfrac{\overline{AB}}{\text{sen}\hat{C}} \Rightarrow \dfrac{\overline{BC}}{\frac{1}{\cancel{2}}} = \dfrac{30}{\frac{\sqrt{2}}{\cancel{2}}} \Rightarrow \overline{BC} = 15\sqrt{2} = 21\text{ km}$

02 D

Sendo o ângulo obtuso 150°, o ângulo agudo é 30°, logo, pela lei dos cossenos:

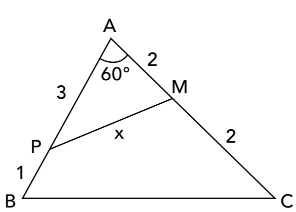
$$\left(\sqrt{3}\right)^2 = x^2 + x^2 - 2\cdot x\cdot x\cdot \cos 30° \Rightarrow 3 = 2x^2 - \cancel{2}x^2.\frac{\sqrt{3}}{\cancel{2}} \Rightarrow$$

$$\Rightarrow 2x^2 - \sqrt{3}x^2 = 3 \Rightarrow \left(2-\sqrt{3}\right)x^2 = 3 \Rightarrow x^2 = \frac{3}{2-\sqrt{3}} \Rightarrow$$

$$\Rightarrow x^2 = 3\left(2+\sqrt{3}\right) \Rightarrow x = \sqrt{3\left(2+\sqrt{3}\right)}$$

03 A

Considere a figura:



I. Se ΔABC é equilátero, o ângulo $\hat{A}$ é 60°.

II. Aplicando a Lei dos Cossenos no triângulo APM, tem-se:

$x^2 = 3^2 + 2^2 - 2 \cdot 3 \cdot 2 \cdot \cos 60°$

$x^2 = 9 + 4 - \cancel{2} \cdot 6 \cdot \frac{1}{\cancel{2}}$

$x^2 = 13 - 6$

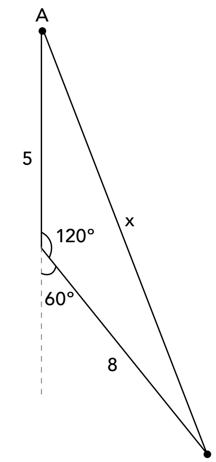
$x = \sqrt{7}$

III. Como se quer o perímetro de APM, tem-se:

$P = 3 + 2 + \sqrt{7} = 5 + \sqrt{7}$

04 B

Aplicando a Lei dos Cossenos:



$x^2 = 5^2 + 8^2 - 2 \cdot 5 \cdot 8 \cdot \cos 120°$

$x^2 = 25 + 64 - 80 \cdot \left(-\frac{1}{2}\right)$

$x^2 = 89 + 40$

$x^2 = 129$

$x = \sqrt{129}$

$x \cong 11\text{ km}$

05 $\dfrac{12}{\text{sen } 60°} = 2r \Rightarrow \dfrac{6}{\frac{\sqrt{3}}{2}} = r$

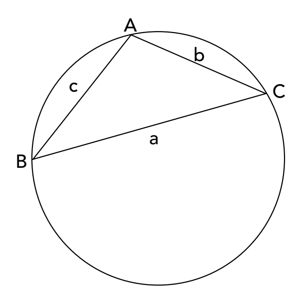
$\sqrt{3}r = 12 \Rightarrow r = \dfrac{12}{\sqrt{3}} \Rightarrow r = 4\sqrt{3}$

ATIVIDADES PROPOSTAS

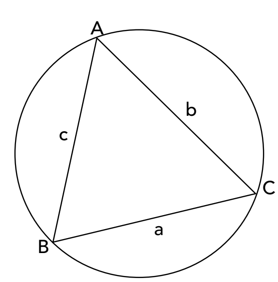
01 C

Para um triângulo inscrito em uma circunferência, vale a lei:

$$\frac{a}{\text{sen }\hat{A}} = \frac{b}{\text{sen }\hat{B}} = \frac{c}{\text{sen }\hat{C}} = 2r$$



I. Tem-se o triângulo a seguir.



a + b + c = 20x

$\text{sen}\,\hat{A} + \text{sen}\,\hat{B} + \text{sen}\,\hat{C} = x$

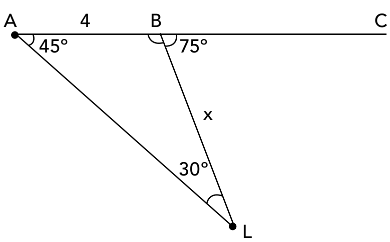
Aplicando a Lei dos Senos, tem-se:

$\frac{a}{\text{sen}\,\hat{A}} = \frac{b}{\text{sen}\,\hat{B}} = \frac{c}{\text{sen}\,\hat{C}} = 2r \Rightarrow$

$\Rightarrow \frac{a+b+c}{\text{sen}\,\hat{A} + \text{sen}\,\hat{B} + \text{sen}\,\hat{C}} = 2r \Rightarrow \frac{20\cancel{x}}{\cancel{x}} = 2r \Rightarrow r = 10$

II. $A = \pi r^2 \Rightarrow A = \pi \cdot 10^2 \Rightarrow A = 100\pi$

02 **C**



$\frac{x}{\text{sen}\,45°} = \frac{4}{\text{sen}\,30°}$

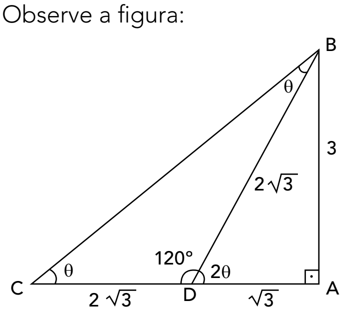
$x \cdot \text{sen}\,30° = 4 \cdot \text{sen}\,45°$

$x \cdot \frac{1}{\cancel{2}} = 4 \cdot \frac{\sqrt{2}}{\cancel{2}}$

$x = \frac{4\sqrt{2}}{1} = 4\sqrt{2}$

03 **D**

I. Observe a figura:



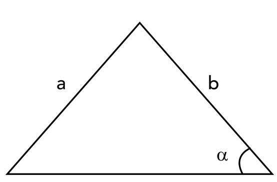
- O ângulo $A\hat{D}B$ é externo do ΔBCD, assim ele é a soma dos internos não adjacentes. Então, tem-se que o ângulo $C\hat{B}D = \theta$.

II. O triângulo BCD é isósceles, pois têm-se dois ângulos congruentes.

III. Para a área do triângulo BCD, tem-se:

$A = \frac{\cancel{2}\sqrt{3} \cdot 2\sqrt{3} \cdot \text{sen}\,120°}{\cancel{2}} \Rightarrow A = \cancel{6}^{\,3} \cdot \frac{\sqrt{3}}{\cancel{2}} \Rightarrow A = 3\sqrt{3}$

04 **C**



I. $3a = 7c$

$a = \frac{7c}{3}$

II. $3b = 8c$

$b = \frac{8c}{3}$

Aplicando a Lei dos Cossenos:

$a^2 = b^2 + c^2 - 2bc \cdot \cos\alpha$

$\left(\frac{7c}{3}\right)^2 = \left(\frac{8c}{3}\right)^2 + c^2 - 2 \cdot \frac{8c}{3} \cdot c \cdot \cos\alpha$

$\frac{49c^2}{9} = \frac{64c^2}{9} + c^2 - \frac{16c^2}{3} \cdot \cos\alpha$

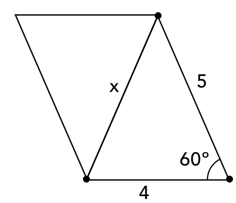
$49c^2 = 64c^2 + 9c^2 - 48c^2 \cdot \cos\alpha$

$48\cancel{c^2} \cdot \cos\alpha = 24\cancel{c^2},\ c \neq 0$

$\cos\alpha = \frac{24}{48}$

$\cos\alpha = \frac{1}{2} \Rightarrow \alpha = 60°$

05 Sendo um dos ângulos do paralelogramo igual a 120°, o outro será 60° e, por ser menor, estará oposto à diagonal menor. Logo:



Pela lei dos cossenos, tem-se:

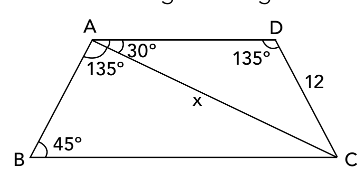
$x^2 = 4^2 + 5^2 - 2 \cdot 4 \cdot 5 \cdot \cos 60° \Rightarrow$

$\Rightarrow x^2 = 16 + 25 - \cancel{2} \cdot 20 \cdot \frac{1}{\cancel{2}} \Rightarrow$

$\Rightarrow x^2 = 41 - 20 \Rightarrow x = \sqrt{21}$

06 **C**

Considere a figura a seguir.



Aplicando a Lei dos Senos no triângulo ACD, tem-se:

$\frac{x}{\text{sen}\,135°} = \frac{12}{\text{sen}\,30°} \Rightarrow \frac{\cancel{2x} \cdot a}{\cancel{2}} \Rightarrow x = 12\sqrt{2}\ \mu.c$

07 **C**

$\frac{\cancel{6}\sqrt{2}}{\text{sen}\,\alpha} = 2 \cdot \cancel{6} \Rightarrow \text{sen}\,\alpha = \frac{\sqrt{2}}{2} \Rightarrow \alpha = 45°$

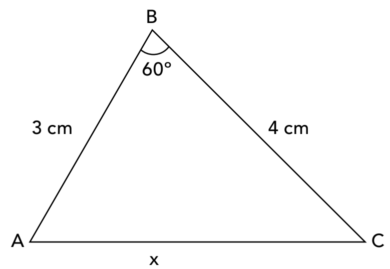
08 **D**

$x^2 = a^2 + a^2 - 2a^2 \cos 120°$

$x^2 = 2a^2 + a^2$

$x = \sqrt{3a^2} = a\sqrt{3}$

09 I. **B**



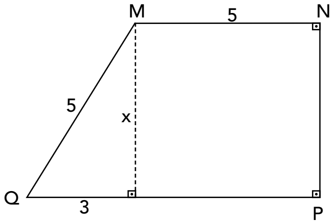
$$x^2 = 3^2 + 4^2 - 2 \cdot 3 \cdot 4 \cdot \cos 60°$$
$$x^2 = 9 + 16 - 24 \cdot \frac{1}{2}$$
$$x^2 = 13 \Rightarrow x = \sqrt{13}$$

II. **B**

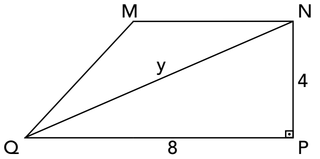
$$6^2 = 4^2 + 5^2 - 2 \cdot 4 \cdot 5 \cdot \cos \varphi$$
$$36 = 16 + 25 - 40 \cos \varphi$$
$$40 \cos \varphi = 5$$
$$\cos \varphi = \frac{1}{8} = 0{,}125$$

10 **A**

I. $x^2 + 3^2 = 5^2$
$x^2 = 25 - 9$
$x^2 = 16$
$x = 4$



$y^2 = 8^2 + 4^2$
$y^2 = 64 + 16$
$y^2 = 80$
$y = 4\sqrt{5}$



II. $\left(4\sqrt{5}\right)^2 = 5^2 + 5^2 - 2 \cdot 5 \cdot 5 \cdot \cos \theta$
$80 = 50 - 50 \cos \theta$
$50 \cos \theta = -30$
$\cos Q\hat{M}N = \cos \theta = -\frac{3}{5}$