Resoluções

Capítulo 12

Cone

ATIVIDADES PARA SALA – PÁG. 6

01 a) $g^2 = 10^2 + 4^2 \Rightarrow g = \sqrt{116} \cong 10{,}7$ cm

b) $A_\ell = \pi rg = 3{,}14 \cdot 4 \cdot 10{,}7 \cong 134{,}4$ cm²

c) $A_b = \pi r^2 = 3{,}14 \cdot 4^2 = 50{,}24$ cm²

$A_t = A_\ell + A_b = 134{,}4 + 50{,}24 = 184{,}64$ cm²

d) Nesse caso, R = g = 10,7 cm e L = 2π · 4 = 8π:

$360° \text{ —— } 2\pi \cdot R \quad \Rightarrow \quad 360° \text{ —— } 2\pi \cdot 10{,}7$

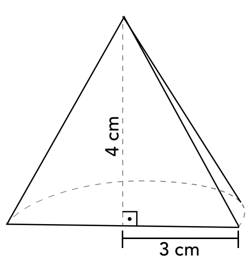
$\alpha \text{ —— } L \qquad\qquad\qquad \alpha \text{ —— } 8\pi$

$\Rightarrow \alpha = \dfrac{8\pi \cdot 360°}{2\pi \cdot 10{,}7} = 134{,}5° = 134°30'$

Logo, α = 134°30'.

02 a)

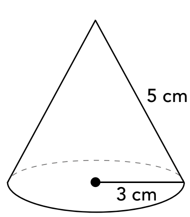
g = 5 cm



$A_\ell = \dfrac{6 \cdot 4}{2} + \dfrac{\pi \cdot 3 \cdot 5}{2} \Rightarrow A_\ell = \dfrac{3(8+5\pi)}{2}$ cm²

$A_t = \dfrac{3(8+5\pi)}{2} + \dfrac{\pi \cdot 3^2}{2} \Rightarrow A_t = \dfrac{3(8+8\pi)}{2}$ cm² $\Rightarrow 12(1+\pi)$ cm²

b)



$A_\ell = \pi \cdot 3 \cdot 5 = 15\pi$ cm²

$A_t = 15\pi + \pi \cdot 3^2 = 24\pi$ cm²

03 $\cancel{\pi} rg + \cancel{\pi} r^2 = 54\cancel{\pi} \Rightarrow 2r^2 + r^2 = 54 \Rightarrow 3r^2 = 54 \Rightarrow$

$r^2 = 18 \Rightarrow r = 3\sqrt{2}$

$h = \dfrac{(2r)\sqrt{3}}{2} \Rightarrow h = r\sqrt{3} \Rightarrow h = 3\sqrt{2} \cdot \sqrt{3}$

$\Rightarrow h = 3\sqrt{6}$ cm

04 $h = \dfrac{g\sqrt{3}}{2} = \sqrt{3} \Rightarrow g = 2 \text{ e } r = 1$

Área da seção $= \dfrac{g^2\sqrt{3}}{4} = \dfrac{2^2\sqrt{3}}{4} = \sqrt{3}$ cm²

05 De acordo com os dados do problema, tem-se α = 72° (ângulo central) e g = R = raio do setor circular = 5 cm. Então:

$A_\ell = \pi R^2 \cdot \dfrac{\alpha}{360} = \pi(5)^2 \cdot \dfrac{\overset{1}{\cancel{72}}}{\underset{5}{\cancel{360}}} = \pi \cdot \overset{5}{\cancel{25}} \cdot \dfrac{1}{\underset{1}{\cancel{5}}} = 5\pi =$

$= 5 \cdot 3{,}14 = 15{,}70$ cm²

ATIVIDADES PROPOSTAS – PÁG. 6

01 $A_\ell = \pi rg \Rightarrow A_\ell = \pi \cdot 5 \cdot 10 \therefore A_\ell = 50\pi$ cm²

$r = 5 \text{ cm} \Rightarrow L = 10 \text{ cm} \Rightarrow 10^2 = r^2 + h^2 \Rightarrow$

$100 = 5^2 + h^2 \Rightarrow h = 5\sqrt{3}$ cm

02 $\dfrac{\cancel{2}r \cdot h}{\cancel{2}} = \pi r^2 \Rightarrow h = \pi r \therefore h = \pi$ m

$A_\ell = \pi rg = \pi \cdot 1 \cdot \sqrt{h^2 + r^2} = \pi\sqrt{\pi^2 + 1}$ m²

03 $\pi \cdot 4^2 \text{ ——— } 360°$

$A_s \text{ ——— } 60°$

$A_s = \dfrac{\cancel{60} \cdot \pi \cdot 16}{\underset{6}{\cancel{360}}} \Rightarrow A_s = \dfrac{8\pi}{3}$ cm²

04 $\alpha = \dfrac{2\pi r}{g} \Rightarrow \dfrac{\cancel{2\pi}}{3} = \dfrac{\cancel{2\pi} \cdot r}{15} \Rightarrow r = 5$ cm

$g^2 = h^2 + r^2 \Rightarrow h = \sqrt{15^2 - 5^2} \Rightarrow h = 10\sqrt{2}$ cm

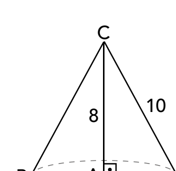
05 $g^2 = h^2 + r^2 \therefore 64 = h^2 + 36 \Rightarrow h = 2\sqrt{7}$ cm

$A_{SM} = \dfrac{2r \cdot h}{2} = r \cdot h = 6 \cdot 2\sqrt{7} = 12\sqrt{7}$ cm²

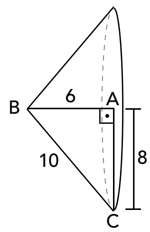
06 $g^2 = 6^2 + 8^2 \therefore g^2 = 100 \therefore g = 10$

a) $A_t = \pi \cdot 6^2 + \pi \cdot 6 \cdot 10$

$A_t = 96\pi$ cm²



b) $A_t = \pi \cdot 8^2 + \pi \cdot 8 \cdot 10$
$A_t = 144\pi \text{ cm}^2$



07 $h = 2\sqrt{21}$ m

$r = 4$ m

$g = \sqrt{(2\sqrt{21})^2 + 4^2} = 10$ m

$\alpha = \frac{2\pi r}{g} = \frac{2\pi \cdot 4}{10} = \frac{4\pi}{5}\text{rad}$

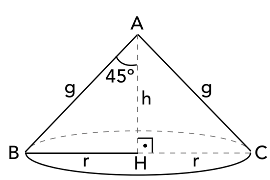
08 $g = 5\sqrt{2}$ cm $\quad h = 7$ cm

$r^2 + h^2 = g^2 \Rightarrow r^2 = 50 - 49 \Rightarrow r = 1$

a) $A_\ell = \pi \cdot 1 \cdot 5\sqrt{2} = 5\pi\sqrt{2} \text{ cm}^2$

b) $A_t = \pi \cdot 1^2 + 5\pi\sqrt{2} = \pi(1 + 5\sqrt{2}) \text{ cm}^2$

09 a) Considere o cone da figura:



No ΔABH retângulo, tem-se:

$\text{sen } 45° = \frac{r}{g} = \frac{\sqrt{2}}{2} \Rightarrow g = r\sqrt{2}$ (I)

Perímetro da seção meridiana:

$2g + 2r = 2 \Rightarrow g + r = 1$ (II)

De (I) e (II), tem-se $r = \sqrt{2} - 1$.

Área total do cone:

$A_t = \pi r \cdot g + \pi r^2 = \pi r(g + r) = \pi(\sqrt{2} - 1) \cdot 1$

$A_t = \pi(\sqrt{2} - 1) \text{ cm}^2$

b) No ΔAHC, tem-se:

$g = r\sqrt{2} \Rightarrow g = (\sqrt{2} - 1)\sqrt{2}$

$g = 2 - \sqrt{2}$

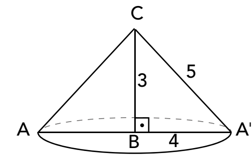
$g^2 = h^2 + r^2 \Rightarrow (2 - \sqrt{2})^2 = h^2 + (\sqrt{2} - 1)^2$

$4 - 4\sqrt{2} + 2 = h^2 + 2 - 2\sqrt{2} + 1$

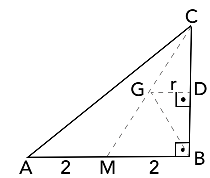
$h^2 = 3 - 2\sqrt{2} \Rightarrow h = \sqrt{3 - 2\sqrt{2}}$ cm

10 a) $A_t = \pi \cdot 4^2 + \pi \cdot 4 \cdot 5$

$A_t = 36\pi \text{ cm}^2$

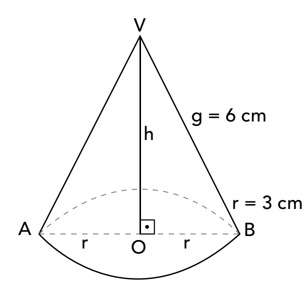


b) $\frac{\overline{GC}}{r} = \frac{\overline{CM}}{2} \Rightarrow \frac{\frac{2}{3}\cancel{\overline{CM}}}{r} = \frac{\cancel{\overline{CM}}}{2} \Rightarrow r = \frac{4}{3}$ cm



ATIVIDADES PARA SALA – PÁG.10

01 A seção meridiana de um cone equilátero é um triângulo equilátero. Como seu perímetro é 18 cm, seu lado mede 6 cm.



Cálculo da altura do cone:

$g^2 = h^2 + r^2 \Rightarrow 36 = h^2 + 9$

$h^2 = 27$

$h = 3\sqrt{3}$ cm

Cálculo do volume:

$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3} \cdot \pi \cdot 3^2 \cdot 3\sqrt{3}$

$V = 9\pi\sqrt{3} \text{ cm}^3$

02 a) V: volume da taça $\Rightarrow V = \frac{1}{3}\pi \cdot 4^2 \cdot 15$

$V = 240 \text{ cm}^3 = 0{,}24$ litros

b) $\Delta ABC \sim \Delta DBE$

$$\frac{\overline{AB}}{\overline{DB}} = \frac{\overline{AC}}{\overline{DE}} \Rightarrow \frac{15}{h} = \frac{4}{r} \Rightarrow r = \frac{4h}{15}$$

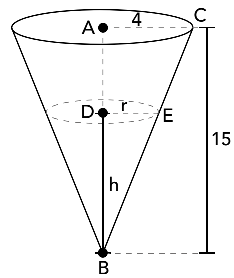
V_1 = volume do suco restante:

$$V_1 = \frac{1}{3}\pi \cdot r^2 \cdot h \Rightarrow$$

$$\Rightarrow \frac{1}{3} \cdot 3 \cdot \left(\frac{4h}{15}\right)^2 \cdot h = \frac{16}{225}h^3$$

Como $V_1 = \frac{V}{2}$, então:

$$\frac{16h^3}{225} = \frac{240}{2} \Rightarrow h = \frac{15\sqrt[3]{4}}{2} \text{ cm}$$



03 $\left.\begin{array}{l} h = 12 \text{ cm} \\ r = 4 \text{ cm} \end{array}\right\} \Rightarrow V_1$: volume do cilindro

$V_1 = \pi \cdot 4^2 \cdot 12 \Rightarrow V_1 = 192\pi \text{ cm}^3$

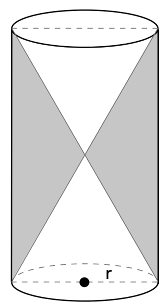
V_2 : volume de cada vaso cônico

$V_2 = \frac{1}{3}\pi \cdot 4^2 \cdot 6 \Rightarrow V_2 = 32\pi \text{ cm}^3$

Assim, o volume procurado é:

$V = V_1 - 2V_2$, ou seja,

$V = 192\pi - 2 \cdot 32\pi \Rightarrow V = 128\pi \text{ cm}^3$



04 **B**

$g = 5$ cm

$2\pi r = 6\pi \Rightarrow r = 3$

$h^2 + r^2 = g^2 \Rightarrow h = 4$ cm

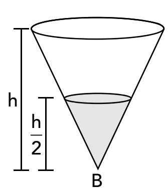
$V = \frac{1}{3} \cdot \pi \cdot 3^2 \cdot 4 = 12\pi$

05 **D**

$$\frac{V}{V_1} = \left(\frac{h}{\frac{h}{2}}\right)^3$$

$$\frac{V}{V_1} = 8 \Rightarrow V_1 = \frac{V}{8}$$

$$\frac{8V}{8} - \frac{V}{8} = \frac{7V}{8}$$



ATIVIDADES PROPOSTAS – PÁG. 10

01 $A_\ell = \pi rg = \cancel{\pi} r \cdot 2r = 24\cancel{\pi} \Rightarrow r = 2\sqrt{3}$

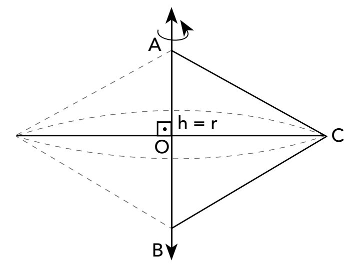
$A_t = \pi rg + \pi r^2 = \pi r \cdot 2r + \pi r^2 = 3\pi r^2 = 3\pi \cdot 12 = 36\pi \text{ cm}^2$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot 12\sqrt{(4\sqrt{3})^2 - (2\sqrt{3})^2} = \frac{1}{3}\pi \cdot 12 \cdot 6 = 24\pi \text{ cm}^3$

02 r: medida do raio do cone = medida da altura do ΔABC (equilátero)

Assim, $r = 6\sqrt{3}$ cm.

L: medida do lado do ΔABC



$$6\sqrt{3} = \frac{L\sqrt{3}}{2}$$

L = 12 cm

Assim, $\overline{AO} = \frac{L}{2} = 6$ cm

V_1: volume do cone $\Rightarrow V_1 = \frac{1}{3} \cdot \pi r^2 \cdot \overline{AO} \Rightarrow$

$\Rightarrow V_1 = \frac{1}{3}\pi(6\sqrt{3})^2 \cdot 6 \Rightarrow V_1 = 216\pi \text{ cm}^3$

V: volume do sólido gerado

$V = 2 \cdot V_1 \Rightarrow V = 432\pi \text{ cm}^3$

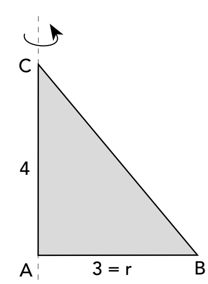
03 $V_{CIL} - 2V_{CONE} = \pi\left(\frac{5\sqrt{3}}{2}\right)^2 \cdot 5 - 2 \cdot \frac{1}{3}\pi \cdot \left(\frac{5\sqrt{3}}{2}\right)^2 \cdot \frac{5}{2} =$

$$\frac{375\pi}{4} - \frac{750\pi}{24} = \frac{1\,500\pi}{24} = \frac{125\pi}{2} \text{ dm}^3$$

04 **B**

Ângulo de rotação	Volume (m^3)
360°	$\frac{1}{3}\pi \cdot 3^2 \cdot 4$
30°	V

$V = \frac{30}{360} \cdot \frac{1}{3} \cdot \pi \cdot 36 \Rightarrow V = \pi \text{ m}^3$



05 **B**

Considerando que os noivos solicitaram o mesmo volume de champanhe em ambas as taças e que a taça redonda é metade de uma esfera, calcula-se:

$$\frac{V_{esfera}}{2} = V_{cone}$$

$$\frac{\frac{4}{3}\pi R^3}{2} = \frac{1}{3}\pi R^2 h$$

$$\frac{\frac{108\pi}{3}}{2} = \frac{9\pi h}{3}$$

$$\frac{108\pi}{6} = \frac{9\pi h}{3}$$

$$\frac{36\pi}{2} = 3\pi h$$

$$6\pi h = 36\pi$$

$$h = 6$$

06 $\frac{2\pi}{3} = \frac{2\pi \cdot 3}{g} \Rightarrow g = 9$ cm

$$g^2 = h^2 + r^2 \Rightarrow h^2 = 81 - 9 = 72 \quad \therefore h = 6\sqrt{2} \text{ cm}$$

$$V = \frac{1}{3}\pi \cdot 3^2 \cdot 6\sqrt{2} = 18\pi\sqrt{2} \text{ cm}^3$$

07 **C**

V_1: volume do vinagre $\Rightarrow V_1 = \frac{1}{3} \cdot \pi \cdot 5^2 \cdot (h - 5)$

V_2: volume do recipiente $\Rightarrow V_2 = \pi \cdot 5^2 \cdot h$

V_3: volume do azeite $\Rightarrow V_3 = V_2 - V_1 =$

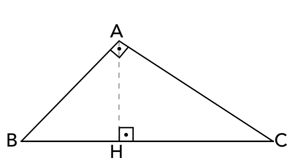
$$= 25\pi h - \frac{25}{3}\pi(h-5) = \frac{25}{3}\pi(2h+5)$$

$$\frac{V_3}{V_1} = 5 \Rightarrow V_3 = 5 \cdot V_1$$

$$\frac{25\pi}{3}(2h+5) = 5\frac{25\pi}{3}(h-5)$$

$h = 10$ cm

08 Aplicando o Teorema de Pitágoras no ΔABC, encontra-se a medida de sua hipotenusa, que corresponde à soma das alturas dos cones.



$$(\overline{BC})^2 = 9^2 + 12^2 \Rightarrow (\overline{BC})^2 = 225 \therefore \overline{BC} = 15 \text{ cm}$$

O raio da base comum aos cones é a medida da altura $\overline{AH}$ do ΔABC.

$$\overline{AB} \cdot \overline{AC} = \overline{BC} \cdot \overline{AH} \Rightarrow 9 \cdot 12 = 15 \cdot \overline{AH} \quad \therefore \overline{AH} = 7{,}2 \text{ cm}$$

Logo, r = 7,2 cm.

O volume V do sólido é a soma $V_1 + V_2$, os quais são, respectivamente, os volumes dos cones de vértices B e C de raio $r = \overline{AH}$; portanto:

$$V = V_1 + V_2 \Rightarrow V = \frac{1}{3} \cdot \pi r^2 \cdot \overline{BH} + \frac{1}{3} \cdot \pi \cdot r^2 \cdot \overline{CH}$$

$$V = \frac{1}{3}\pi r^2 \underbrace{(\overline{BH} + \overline{CH})}_{\overline{BC}}$$

$$V = \frac{1}{3}\pi r^2 \cdot \overline{BC}$$

Substituindo numericamente, obtém-se:

$$V = \frac{1}{3} \cdot \pi \cdot 7{,}2^2 \cdot 15 \therefore V \cong 814 \text{ cm}^3$$

09 **C**

r: raio da base do cone

h: altura do cone $\frac{r}{h} = k_{(\text{constante})}$

V_1: volume inicial do monte de minério

$$V_1 = \frac{1}{3}\pi r^2 h$$

$$h_1 = 1{,}3h \Rightarrow r_1 = 1{,}3r$$

V_2: volume final do monte de minério

$$V_2 = \frac{1}{3}\pi r_1^2 \cdot h_1 = \frac{1}{3}\pi(1{,}3r)^2 \cdot 1{,}3h =$$

$$= 2{,}197\left(\frac{1}{3}\pi r^2 h\right) = 2{,}197V_1$$

Assim: $V_2 - V_1 = 1{,}197 \cdot V_1 = 119{,}7\% \cdot V_1 \cong 120\% \cdot V_1$.

10 **E**

$$(r, h, g) = (r, r+4, r+8)$$

$$(r+8)^2 = r^2 + (r+4)^2$$

$$\cancel{r^2} + 16r + 64 = \cancel{r^2} + r^2 + 8r + 16$$

$$r^2 - 8r - 48 = 0 \Rightarrow r = \frac{8 \pm 16}{2} \begin{cases} r = 12 \\ r = -4 \text{ (não convém)} \end{cases}$$

$$V = \frac{1}{3}\pi \cdot 12^2 \cdot 16 = 768\pi \text{ dm}^3$$