Resoluções

Resposta da atividade da página 5:

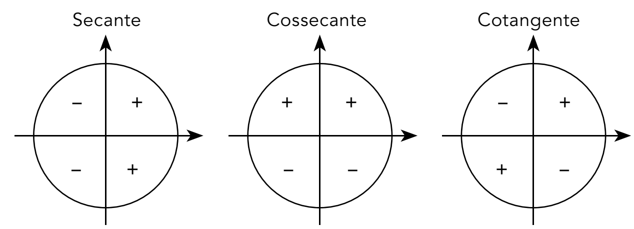
	arco	x°	sen (x)	cos (x)	tg (x)	cotg (x)	sec (x)	cossec (x)
1º quadrante	0	0°	0	1	0	não existe	1	não existe
	$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
	$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
	$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
	$\frac{\pi}{2}$	90°	1	0	não existe	0	não existe	1
2º quadrante	$\frac{2\pi}{3}$	120°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	–2	$\frac{2\sqrt{3}}{3}$
	$\frac{3\pi}{4}$	135°	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	–1	–1	$-\sqrt{2}$	$\sqrt{2}$
	$\frac{5\pi}{6}$	150°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$\frac{-2\sqrt{3}}{3}$	2
	π	180°	0	–1	0	não existe	–1	não existe
3º quadrante	$\frac{7\pi}{6}$	210°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{-2\sqrt{3}}{3}$	–2
	$\frac{5\pi}{4}$	225°	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
	$\frac{4\pi}{3}$	240°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	–2	$\frac{-2\sqrt{3}}{3}$
	$\frac{3\pi}{2}$	270°	–1	0	não existe	0	não existe	–1
4º quadrante	$\frac{5\pi}{3}$	300°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	2	$\frac{-2\sqrt{3}}{3}$
	$\frac{7\pi}{4}$	315°	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	–1	–1	$\sqrt{2}$	$-\sqrt{2}$
	$\frac{11\pi}{6}$	330°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	–2
	2π	360°	0	1	0	não existe	1	não existe

Capítulo 8

Relações trigonométricas – Secante, cossecante e cotangente de um arco trigonométrico

ATIVIDADES PARA SALA

01 Sabendo que:



Tem-se:

a) $\sec 300° = \sec 60° = \dfrac{1}{\cos 60°} = \dfrac{1}{\frac{1}{2}} = 2$

b) $\sec \dfrac{5\pi}{4} = \sec 225° = \dfrac{1}{-\cos 45°} = \dfrac{1}{-\frac{\sqrt{2}}{2}} = -\sqrt{2}$

c) $\text{cossec } 330° = -\text{cossec } 30° = -\dfrac{1}{\text{sen } 30°} = -\dfrac{1}{\frac{1}{2}} = -2$

d) $\text{cossec } \dfrac{8\pi}{3} = \text{cossec } 480° = \text{cossec } 120° =$

$= \dfrac{1}{\text{sen } 60°} = \dfrac{1}{\frac{\sqrt{3}}{2}} = \dfrac{2\sqrt{3}}{3}$

e) $\text{cotg } 240° = \text{cotg } 60° = \dfrac{1}{\text{tg } 60°} = \dfrac{\sqrt{3}}{3}$

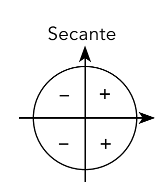
f) $\text{cotg}\left(-\dfrac{7\pi}{6}\right) = \text{cotg }(-210°) = -(\text{cotg } 210°) =$

$= -\text{cotg } 30° = -\sqrt{3}$

02 $y = \dfrac{\cos 360° + 2\text{tg } 45° - \text{sen } 180°}{\text{cotg } 90° \cdot \text{cossec } 90° + \sec 720°} = \dfrac{1+2-0}{1} = 3$

03 a) $\sec \dfrac{11\pi}{6} = \sec 330° =$ (+) (4º quadrante)

b) $\sec \dfrac{7\pi}{4} = \sec 315° =$ (+) (4º quadrante)



c) $\sec\dfrac{7\pi}{3} = \sec 420° = \sec(420° - 360°) =$

$= \sec 60° = +$ (1º quadrante)

d) $\sec\dfrac{3\pi}{4} = \sec 135° = -$ (2º quadrante)

04 $\left(\operatorname{cossec}\dfrac{\pi}{3} + \operatorname{sen}\dfrac{\pi}{3}\right)\cdot\left(\operatorname{sen}\dfrac{\pi}{3} - \sec\dfrac{\pi}{6}\right) =$

$= \left(\dfrac{2\sqrt{3}}{3} + \dfrac{\sqrt{3}}{2}\right)\cdot\left(\dfrac{\sqrt{3}}{2} - \dfrac{2\sqrt{3}}{3}\right) =$

$= \left(\dfrac{4\sqrt{3}+3\sqrt{3}}{6}\right)\cdot\left(\dfrac{3\sqrt{3}-4\sqrt{3}}{6}\right) =$

$= \dfrac{7\sqrt{3}}{6}\cdot\left(-\dfrac{\sqrt{3}}{6}\right) = -\dfrac{21}{36} = -\dfrac{7}{12}$

05 Pela condição dada, tem-se:

$\sec x = \dfrac{2k-1}{k} \Rightarrow \cos x = \dfrac{k}{2k-1}$

Se $x \in \left]\dfrac{-\pi}{2}, \dfrac{3\pi}{2}\right] \Rightarrow -1 \le \cos x \le 1$

$-1 \le \dfrac{k}{2k-1} \le 1 \Rightarrow \dfrac{k}{2k-1} \ge -1$ (I) e $\dfrac{k}{2k-1} \le 1$ (II)

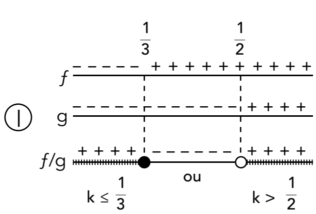
Resolvendo (I), tem-se:

$\dfrac{k}{2k-1} + 1 \ge 0 \Rightarrow \dfrac{3k-1}{2k-1} \ge 0$

$f(x) = 3k - 1 \Rightarrow 3k - 1 = 0 \Rightarrow k = \dfrac{1}{3}$

$g(x) = 2k - 1 \Rightarrow 2k - 1 = 0 \Rightarrow k = \dfrac{1}{2}$

Observação: $k \neq \dfrac{1}{2}$



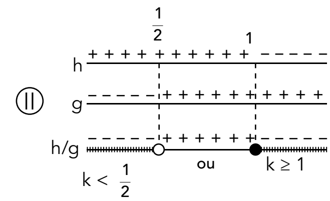
Resolvendo (II), tem-se:

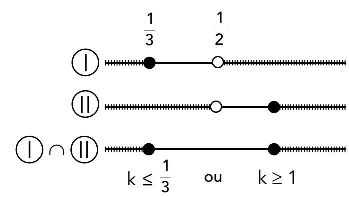
$\dfrac{k}{2k-1} - 1 \le 0 \Rightarrow \dfrac{1-k}{2k-1} \le 0$

$h(x) = 1 - k \Rightarrow 1 - k = 0 \Rightarrow k = 1$

$g(x) = 2k - 1 \Rightarrow 2k - 1 = 0 \Rightarrow k = \dfrac{1}{2}$

Observação: $k \neq \dfrac{1}{2}$





$S = \left\{\forall k \in \mathbb{R} \,/\, k \le \dfrac{1}{3} \text{ ou } k \ge 1\right\}$

ATIVIDADES PROPOSTAS

01 **F, V, F, V, F, V**

(F) $\operatorname{cossec} 45° = \sqrt{2}$

(V)

(F) $\sec\dfrac{\pi}{2} = \nexists$

(V)

(F) $\operatorname{cotg}\dfrac{\pi}{6} = \sqrt{3}$

(V)

02 $\dfrac{4}{3k-1} = \dfrac{2}{3} \Rightarrow 6k - 2 = 12 \Rightarrow k = \dfrac{7}{3}$

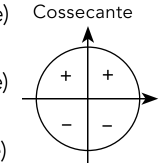
03 a) $\operatorname{cossec}\dfrac{2\pi}{3} = \operatorname{cossec} 120° = +$ (2º quadrante)

b) $\operatorname{cossec}\dfrac{5\pi}{3} = \operatorname{cossec} 300° = -$ (4º quadrante)

c) $\operatorname{cossec}\dfrac{7\pi}{4} = \operatorname{cossec} 315° = -$ (4º quadrante)

d) $\operatorname{cossec}\dfrac{7\pi}{6} = \operatorname{cossec} 210° = -$ (3º quadrante)

Cossecante



04 $\dfrac{3m^2 - 3}{4m} = \sec 60°$

$\dfrac{3m^2 - 3}{4m} = 2$

$3m^2 - 8m - 3 = 0$

$\Delta = 64 + 36 = 100$

$m = \dfrac{8 \pm 10}{6} \Rightarrow m' = 3 \text{ ou } m'' = -\dfrac{1}{3}$

05 **D**

$\sec 10\pi = \sec 1\,800 = 1$

06 **E**

$$\text{sen}\, 2\pi - \sec\frac{4\pi}{3} - \text{cossec}\,\frac{5\pi}{6} \Rightarrow$$
$$0-(-2)-(2)=2-2=0$$

07 $\sec 70° = \text{cossec}\, 20° = \dfrac{1}{\text{sen}\, 20°} = \dfrac{1}{\frac{7}{20}} = \dfrac{20}{7}$

08 $\text{cossec}\, 2\,220° + \sec 4\,020° = \dfrac{A}{3}$

$$\text{cossec}\, 2\,220° = \text{cossec}\, 60° = \frac{1}{\text{sen}\, 60°} = \frac{2\sqrt{3}}{3}$$

$$\sec 4\,020° = \sec 60° = \frac{1}{\cos 60°} = 2$$

Logo,

$$\frac{2\sqrt{3}}{3} + 2 = \frac{A}{3} \Rightarrow \frac{2\sqrt{3}+6}{\cancel{3}} = \frac{A}{\cancel{3}}$$
$$A = 2\cdot(\sqrt{3}+3)$$

09 $m - \sqrt{3}\cdot \text{cotg}\, x = -1$

$$\text{cotg}\, x = \frac{m+1}{\sqrt{3}} \Rightarrow \text{tg}\, x = \frac{\sqrt{3}}{m+1}$$

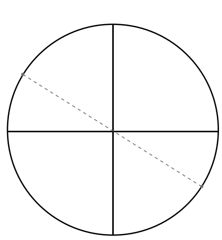
$$\text{tg}\, x \in \left]\frac{\sqrt{3}}{3}, \sqrt{3}\right[\Rightarrow \frac{\sqrt{3}}{3} < \text{tg}\, x < \sqrt{3} \Rightarrow \frac{\sqrt{3}}{3} < \frac{\sqrt{3}}{m+1} < \sqrt{3}$$

$$\frac{1}{3} < \frac{1}{m+1} < 1 \Rightarrow m+1 < 3 < 3m+3 \Rightarrow \begin{cases} m<2 \\ 0<m \end{cases} \Rightarrow 0<m<2$$

10 Pela condição dada, tem-se:

$\text{cotg}\, x = -5 \;\therefore\; \text{tg}\, x = -\dfrac{1}{5}$

$x \in [-4\pi; 4\pi]$



Note que, no intervalo -4π a 4π, são dadas 4 voltas no ciclo.
Como, a cada volta, têm-se duas raízes, ao todo há 8 raízes.